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Bending Behavior of Simply Supported Skew Plates

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Abstract: This paper presents the deflection studies made on skew plates subjected to uniformly distributed load/concentrated load for simply supported boundary conditions using MSC/NASTRAN. The CQUAD4 and CQUAD8 elements of MSC/NASTRAN are validated against literature values. The CQUAD8 element has been found to yield better results compared with the CQUAD4 element and hence it used in the present studies. The variation of deflection for isotropic skew plates with aspect ratio and length to thickness ratio are presented. The deflections are found to decrease with increase in the skew angle.

Keywords: Skew Plates, Finite Element, Bending Analysis, Non-Dimensional Deflection Coefficients.

1 INTRODUCTION

The skew plates find wide range of application in civil, marine, aeronautical and mechanical engineering applications. They are often used in modern structures in spite of the mathematical difficulties involved in their study. The various applications of skew plates can be found in swept wings of aero planes, complex alignment problems in bridge design, ship hulls and parallelogram slabs in buildings. The exact solutions to skew plate bending problems are rare, and those available in the literature are based on approximate methods.

Over the past four decades, a lot of research has been focused on the static and dynamic analysis of skew plates. A good portion of this work was devoted to static analysis of skew plates using either an analytical or numerical solution procedure. Morley (1963) [1] has presented a very thorough overview of the analytical solution methods. Analytical solution procedure requires use of a series like trigonometric series, power series , polynomial series , complex series, bi-harmonic eigen-functions and Fourier series which have been employed in one form or another for the analysis of skew plates[3-10].However, when the analytical methods fail to provide solutions to the problem, the numerical techniques such as finite-difference (FDM)[11-13], finite-element(FEM)[14-26] ,finite-strip element method(FSM)[27-30] and differential quadrature methods(DQM)[31], become the most commonly used procedures for the analysis of skew plates. In addition, various other techniques have also been used for the analysis of skew plates. Variational solution [1-3,6,10], electrical analogy [32-33], point matching [34-35], conformal mapping [36-37], and equivalent grid method [38] have been used.

Today the skew plate problem has been widely used by FEM as a benchmark check on the capability of a particular newly developed finite element. This paper deals with the studies on bending behaviour of skew plates using CQUAD4 and CQUAD8 elements of MSC/NASTRAN. The accuracy of the elements has been verified with literature values. The effects of skew angle, aspect ratio and length to thickness ratios on the non-dimensional central deflection coefficient (W_f) of skew plate are investigated in the present study.

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2 CONVERGENCE AND VALIDATION STUDIES

2.1 Convergence Study

The geometry of the skew plate with global and local coordinate systems is shown in the Figure 1 in which *u* and *v* are the displacement variables in *x* and *y* directions respectively. Since *u* and *v* are inclined to the skew edges, the displacement boundary conditions cannot be applied directly. Hence a local coordinate system (x', y') normal and tangential to the skew edges is chosen.

To obtain accurate and reliable results, it is necessary to study the convergence of the results so as to establish the optimum number of elements required in the finite element model. The convergence study has been performed on simply supported(S-S-S-S) (S3) [39] skew plates having aspect ratio (=a/b) of 1.0 and skew angles 0^o, 15^o, 30^o and 45^o using CQUAD4 (four-noded) and CQUAD8 (eight-noded iso-parametric curved shell element) elements of MSC / NASTRAN. The convergence details are furnished in Table 1.

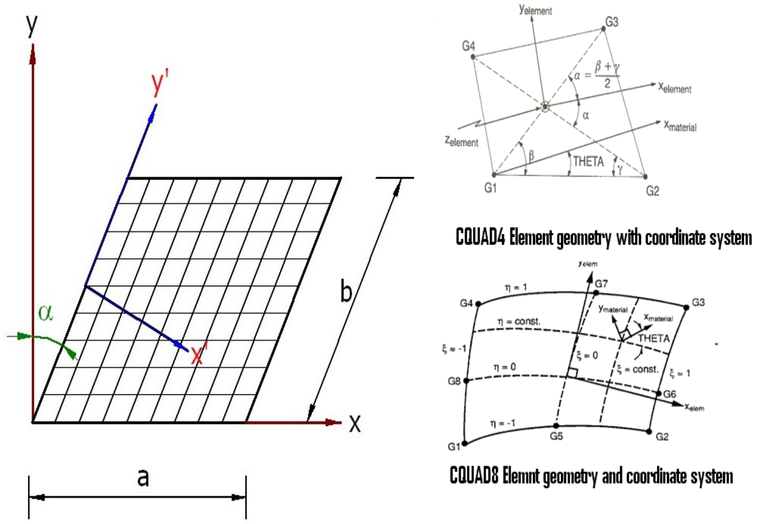

Figure 1: Global and Local Coordinate Systems for the Skew Plate with Finite Element Mesh and Elements Coordinate system.

Table 1: Convergence Study for Non dimensional central deflection (W_f) X10^{-3} for Simply Supported Skew Plates (a/b=1, a/t =100 and μ=0.3)

Authors		**Concentrated load**				**Uniformly distributed load**			
		Skew Angle (α)				**Skew Angle (α)**			
		0°	**15°**	**30°**	**45°**	**0°**	**15°**	**30°**	**45°**
Present (16X16)	CQUAD4	5.7637	5.4139	4.4395	3.0586	1.2928	3.6538	2.5549	1.2862
	CQUAD8	5.6318	5.2893	4.3351	2.9816	1.2681	3.6355	2.5329	1.2479
Present (18X18)	CQUAD4	5.7454	5.3974	4.4267	3.0531	1.2880	3.6501	2.5549	1.2879
	CQUAD8	5.6336	5.2912	4.3388	2.9871	1.2681	3.6373	2.5366	1.2538
Present (20X20)	CQUAD4	**5.7301**	**5.3827**	**4.4175**	**3.0476**	**1.2844**	**3.6483**	**2.5549**	**1.2893**
	CQUAD8	**5.6355**	**5.2930**	**4.3406**	**2.9926**	**1.2880**	**3.6373**	**2.5402**	**1.2586**
Morley(1963)		-	**5.276**	**4.330**	**2.970**	-	-	**2.560**	-
Argyris (1965)		-	-	-	-	-	-	-	**1.300**
Ganga Rao et al.(1988)		-	-	-	-	-	**3.638**	**2.560**	**1.320**
Butalia et al.(1990)		-	**5.238**	**4.250**	**2.870**	-	**3.624**	**2.489**	**1.195**
A.R Krishna Reddy(1995)		-	**5.258**	**4.281**	**2.920**	-	**3.620**	**2.536**	**1.296**

2.2 Validation Check

The validation for the present elements is performed by comparing the values for the Non dimensional central deflection coefficient (W_f) obtained in this work with those available in the literature and is presented in Tables 2 and 3 for simply supported(S-S-S-S) isotropic skew plates under concentrated and uniformly distributed loads. It can also be observed that the results obtained from both elements are compared with those values which are available in the literature. The results indicate that the deflections obtained from CQUAD4 solution are exceeding the solution marginally than the literature once *(about 2%)*. But the results obtained from CQUAD8 elements are in close agreement with values from the literature, up to the second decimal. Based on this conclusion, the present work is continued with the use of CQUAD8 elements for some other useful studies on skew plates

Table 2: Non dimensional central deflection for simply supported Isotropic skew plate subjected to concentrated load (a/b=1, a/t=100 and μ=0.3)

Researchers	**Non dimensional central deflection (W_f) X10^{-3}**			
	Skew angle(α)			
	0°	**15°**	**30°**	**45°**
Morley(1963)	-	5.276	4.330	2.970
Butalia et al.(1990)	-	5.238	4.250	2.870

Krishna Reddy(1995)	-	5.258	4.281	2.920
Present	**5.632**	**5.293**	**4.340**	**2.991**

Table 3: Non dimensional central deflection for simply supported Isotropic skew plate subjected to uniformly distributed load (a/b=1, a/t=100 and μ=0.3)

Researchers	**Non dimensional central deflection (W_f) X10^{-3}**			
	Skew angle(α)			
	0^{O}	**15^{O}**	**30^{O}**	**45^{O}**
Morley(1963)	-	-	2.560	-
Argyris (1965)	-	-	-	1.300
Sampath et. al(1966)	-	-	2.563	1.327
Iyengar et al.(1971)	-	-	2.596	1.361
Rajaiah et. al(1974)	-	-	2.560	1.317
Ganga Rao et al.(1988)	-	3.638	2.560	1.320
Butalia et al.(1990)	-	3.624	2.489	1.195
Harutoshi et.al(1995)	-	-	2.560	1.317
Krishna Reddy(1995)	-	3.620	2.536	1.296
Present	**4.053**	**3.637**	**2.556**	**1.258**

3 RESULTS AND DISCUSSIONS

In the present study, simply supported (S-S-S-S) skew plate under uniformly distribute load (UDL) and concentrated load have been considered. The transverse displacements (W_f) are presented in the form of non- dimensional coefficient defined as follows:

$$\text{Isotropic plate with UDL } = W_f = \frac{wD}{qa^4} \quad (1)$$

$$\text{Isotropic plate with concentrated load } = W_f = \frac{wD}{Qa^2} \quad (2)$$

A simply supported skew plate having all edges are simply supported for various skew angles (α) under uniformly distributed as well as central concentrated load is analyzed. The non dimensional central displacements are evaluated using the finite element scheme. A comparison of the same with that of the literature values of Morley (1963), Butalia et al. (1990), Krishna Reddy (1995) in respect of normal displacement are in good agreement and are presented in Table 4.

A perusal of the Table 4 reveals the following. For lower skew angle (α=15^{o}) the present values are lower by 0.5% of the analytical results of (Ganga Rao and Chaudhary, 1988) and finite element results of (Butalia et al.,1990). For skew angle of 30^{o} ,the present values are lower than the analytical results (Morley , 1993 and Ganga Rao and Chaudhary , 1988) by 1% and higher than the FEM values of Butalia et al.(1990) by about 1.8%.For α=45^{o} the percentage difference increase. The present solution gives about 2% lower values in comparison with the FEM values of Butalia et al. (1990). The values obtained by the present FEM method, using the eight noded quadratic plate elements, are always closer to analytical values, than that of the Butalia et al. (1990).

Table 4: Non dimensional central deflection (W_f) X10^{-3} Simply Supported Isotropic Skew Plates

a/b	a/t	Non-dimensional central deflection (W_f) X10^{-3}							
		concentrated Load				Uniformly distributed load			
		Skew angle(α)							
		0^{o}	15^{o}	30^{o}	45^{o}	0^{o}	15^{o}	30^{o}	45^{o}
0.5	1000	7.219	6.730	5.356	3.548	10.120	8.950	6.038	2.840
	500	7.229	6.734	5.388	3.549	10.124	8.955	6.039	2.846
	100	7.240	6.753	5.413	3.572	10.130	8.960	6.042	2.849
	50	7.311	6.822	5.479	3.635	10.140	8.969	6.051	2.851
	20	7.789	7.292	5.921	4.041	10.210	9.040	6.105	2.889
1.0	1000	5.604	5.267	4.315	2.967	4.050	3.630	2.541	1.253
	500	5.611	5.268	4.316	2.969	4.053	3.634	2.542	1.254
	100	5.632	5.293	4.340	2.991	4.058	3.637	2.556	1.258
	50	5.708	5.367	4.412	3.058	4.060	3.643	2.571	1.264
	20	6.234	5.881	4.903	3.513	4.110	3.684	2.616	1.294

	1000	3.129	2.927	2.368	1.585	1.524	1.358	0.936	0.456
	500	3.129	2.929	2.369	1. 601	1.525	1.359	0.937	0.457
1.5	100	3.155	2.952	2.392	1.612	1.526	1.360	0.938	0.458
	50	3.231	3.028	2.465	1.682	1.530	1.364	0.941	0.461
	20	3.767	3.555	2.970	1.924	1.550	1.389	0.984	0.477
	1000	1.807	1.686	1.345	0.901	0.632	0.559	0.377	0.177
	500	1.808	1.689	1.355	0.902	0.633	0.567	0.378	0.178
2.0	100	1.833	1.711	1.379	0.924	0.634	0.563	0.379	0.179
	50	1.910	1.787	1.452	0.992	0.636	0.564	0.380	0.181
	20	2.440	2.316	1.958	1.459	0.654	0.580	0.394	0.190
	1000	1.158	1.080	0.867	0.576	0.294	0.258	0.171	0.016
	500	1.159	1.082	0.868	0.577	0.295	0.258	0.172	0.017
2.5	100	1.184	1.105	0.892	0.592	0.296	0.259	0.173	0.018
	50	1.261	1.182	0.965	0.667	0.297	0.261	0.174	0.019
	20	1.798	1.711	1.470	0.814	0.308	0.272	0.182	0.022

Further, for isotropic skew plates of skew angle α=0°,15°,30° and 45°, aspect ratio 0.5 to 2.5 and relative thickness has been carefully investigated and are presented in Table 4 in the form of non dimensional coefficients (W_f). The deflection profile for various skew angles along the axis 'x' is presented in Figure 2 and 3 for simply supported under concentrated load and uniformly distributed loads respectively.

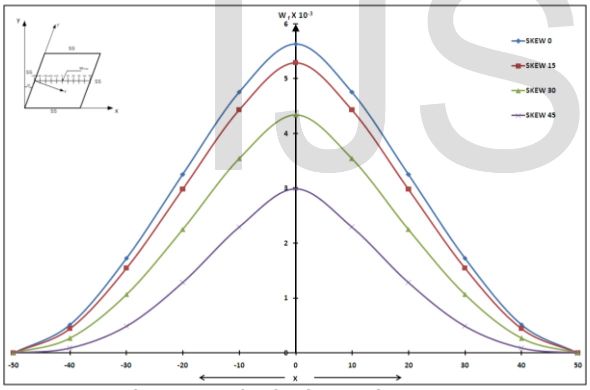

Figure 2: Deflection Profile for Simply Supported Isotropic Skew Plate Under Concentrated Load

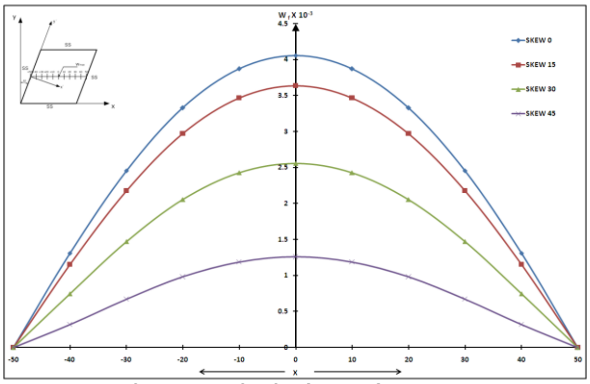

Figure 3: Deflection Profile for Simply Supported Isotropic Skew Plate under uniformly distributed Load

4 CONCLUSIONS

It has been observed that the present finite element model with eight noded isoparametric elements performs excellently, even for large angles of skew for both uniformly distributed and concentrated loading. In the present study, the range of skew angle considered is up to 45°. This is due to the reason that the skew plated structures in reality have skew angles or equal to 45° mostly.

The numerical results displayed for different skew angles and support conditions will not only establish the effectiveness of the present eight noded iso-parametric elements but also provide a ready reference for the future researcher in this area using eight-noded iso-parametric element of MSC/NASTRAN.

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NOMENCLATURE

a	:	Plate length
b	:	Plate width
t	:	Plate thickness
E	:	Modulus of elasticity
D	:	Plate bending rigidity, $Et^3/12(1-\nu^2)$
q	:	Uniformly distributed load
Q	:	Concentrated load
W	:	Deflection

W_f : Non-dimensional deflection coefficient
α : Skew angle
ν : Poisson's ratio

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